CS111

The existence of multiple positive solutions of a Riemann-Liouville fractiona q-difference equation under four-point boundary value condition with p-Laplacian operator

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This paper mainly studies the existence of multiple positive solutions of a class of Riemann-Liouville fractional q-difference equations under the four-point boundary value condition with p-Laplacian operator. The existence of two positive solutions of th e q-difference equation is verified by the monotonic iterative m-ethod. Finally, an example is used to prove the validity of the ma in results obtained.

Introduction

The calculus invented by Newton and Leibnitz is the watershed between modern mathematics and ancient mathematics. Fractional calculus is a related theory about differentiation and integration of any order. It is the extension of integer-order calculus. From q-differential calculus and quantum after the calculus was proposed by Jackson, it attracted the attention of many scholars to the q-difference equation. Quantum calculus is called infinite calculus. It replaces the classical derivative with a difference operato rand can be used to calculate non-differentiable functions. In addition to the application of q-difference to orthogonal polynomials, combinatorics, hypergeometric functions and other mathematical fields, q-differences are inreasingly used in natural sciences and engineering.

We discuss the following equation:

$$\begin{cases} D_q^{\alpha}\left(\phi_p\left(D_q^{\beta}x\right)(t)\right)+h(t,x(t))=0, \quad 0<t<1, \\ x(0)=0, \quad x(1)=aD_q^{\gamma}x(\xi), \quad D_q^{\beta}x(0)=0, \quad D_q^{\beta}x(1)=bD_q^{\beta}x(\eta), \end{cases}$$

where $h\in C([0,1]\times[0,+\infty),[0,+\infty))$, D_q^{α}, D_q^{β} and D_q^{γ} stand for the Riemann-Liouville fractional q-derivative, φ_p is p-Laplacian operator, $p>1$, $\varphi_p(s)=|s|^{p-1}s$, $\varphi_p^{-1}=\varphi_{q^*}$,

$\frac{1}{p}+\frac{1}{q^*}=1, 1<\alpha,\beta\leq 2, \gamma=\frac{\beta-1}{2}, 0<\xi\leq\frac{1}{2}, 0<\eta<1, a,b\in[0,+\infty), a\Gamma_q(\beta)\xi^{\frac{\beta-1}{2}}<\Gamma_q\left(\frac{\beta+1}{2}\right)$,

$b^{p-1}\eta^{\alpha-1}<1$.

Example

$$\begin{cases} D_{\frac{1}{2}}^{\frac{3}{2}}\left(\phi_{\frac{3}{2}}\left(D_{\frac{1}{2}}^{\frac{3}{2}}x(t)\right)\right)=\frac{x^2}{15}+\frac{tx}{12}, \quad 0<t<1 \\ x(0)=0, \quad x(1)=\frac{1}{4}D_{\frac{1}{2}}^{\frac{1}{4}}x\left(\frac{1}{2}\right), \quad D_{\frac{1}{2}}^{\frac{3}{2}}x(0)=0, \quad D_{\frac{1}{2}}^{\frac{3}{2}}x(1)=\frac{1}{2}D_{\frac{1}{2}}^{\frac{3}{2}}x\left(\frac{1}{2}\right). \end{cases}$$

Where $q=\frac{1}{2}$, $p=\frac{3}{2}$, $h(y,x(t))=\frac{x^2}{15}+\frac{tx}{12}$, $\alpha=\beta=\frac{3}{2}$, $\gamma=\frac{1}{4}$, $\xi=\frac{1}{2}$, $\eta=\frac{1}{2}$, $a=\frac{1}{4}$, $b=\frac{1}{2}$.